Comparing interest rates

1.4

In the last section we transposed the compound interest formula to find the principal, the annual interest rate and the number of years for an investment. We will use these transposition skills to compare the amount of interest earned in the first year of an investment under various compounding periods.

We will make these comparisons using a principal of \$10 000 and an annual interest rate of 6%.

If the compounding period is 1 year, then $A = 10\,000(1 + 0.06)^1 = 10\,600$. However, we are interested only in the amount of interest earned, which is calculated using $I = A - P$. In this case this gives \$600 interest.

If the compounding period is 6 months, then $A = 10\,000(1 + 0.03)^2 = 10\,609$, which represents \$609 interest. So, the percentage return is:

$$\frac{609}{10\,000} \times 100 = 6.09\%$$

The following table summarises this information for a number of compounding periods, and includes the simple interest calculation as well. This serves to remind us that simple interest over 1 year is the same as compound interest, compounded annually, for 1 year.

Case	Number of compounding periods	Interest in the first year	Percentage return
A	0 (simple interest)	\$600	$\frac{600}{10\,000} \times 100 = 6\%$ p.a.
B	1 (annual)	\$600	$\frac{600}{10\,000} \times 100 = 6\%$ p.a.
C	2 (6 monthly)	\$609	$\frac{609}{10\,000} \times 100 = 6.09\%$ p.a.
D	4 (quarterly)	\$613.64	$\frac{613.64}{10\,000} \times 100 \approx 6.14\%$ p.a.
E	12 (monthly)	\$616.78	$\frac{616.78}{10\,000} \times 100 \approx 6.17\%$ p.a.

In each of these cases the **nominal interest rate** is 6% p.a. but we can see the amount of interest earned is not a constant. To help make comparisons easier we like to calculate the **effective interest rate.** In case C above this is 6.09% p.a., in Case D it is 6.14% p.a. and for case E it is 6.17% p.a. This is the rate that would give the amount if simple interest was applied for 1 year.

$$R_{ef} = \left(\frac{I_1}{P}\right) \times 100$$

where R_{ef} = effective interest rate or real interest rate (p.a.) as a percentage
I_1 = amount of interest after the first year
P = principal

We can see that the more frequently we compound the interest, the higher the effective interest rate.

Worked Example 8

WE 8

Find the effective rate of interest (R_{ef}) if \$25 000 is invested for 1 year at the following rates. Where necessary round your answers to three decimal places.

(a) a simple rate of 9.5% p.a.

(b) an annually compounding rate of 9.5% p.a.

(c) a monthly compounding rate of 9.5% p.a.

(d) a weekly compounding rate of 9.5% p.a.

Thinking	Working
(a) For simple interest, the effective rate is the same as the nominal rate.	(a) $R_{ef} = 9.5\%$ p.a.
(b) For annual compounding, the effective rate is the same as the nominal rate.	(b) $R_{ef} = 9.5\%$ p.a.
(c) 1 Calculate the total amount after the first year. Record your answer to four decimal places.	(c) $A = P(1+r)^n$ $= 25\,000\left(1 + \frac{0.095}{12}\right)^{12}$ $= 27\,481.1896$
2 Calculate the amount of interest earned in the first year.	$I = A - P$ $= 27\,481.1896 - 25\,000$ $= 2481.1896$
3 Calculate the effective interest rate.	$R_{ef} = \frac{I}{P} \times 100$ $= \frac{2481.1896}{25\,000} \times 100$ $= 9.9247$
4 Round your answer to three decimal places.	The effective interest rate is 9.925% p.a.
(d) 1 Calculate the total amount after the first year. Record your answer to four decimal places.	(d) $A = P(1+r)^n$ $= 25\,000\left(1 + \frac{0.095}{52}\right)^{52}$ $= 27\,489.0887$
2 Calculate the amount of interest earned in the first year.	$I = A - P$ $= 27\,489.0887 - 25\,000$ $= 2489.0887$
3 Calculate the effective interest rate.	$R_{ef} = \frac{I}{P} \times 100$ $= \frac{2489.0887}{25\,000} \times 100$ $= 9.9563$

4 Round your answer to three decimal places.	The effective interest rate is 9.956% p.a.

The effective interest rate is useful for comparing situations with different nominal rates and different compounding periods.

A formula for converting nominal rates to effective rates

A simple formula for converting nominal interest rates to effective interest rates is shown below. We multiply by 100 at the end to convert the rate to a percentage.

$$R_{ef} = \left[\left(1 + \frac{R}{n}\right)^n - 1\right] \times 100$$

where R_{ef} = effective interest rate (p.a.) as a percentage
R = nominal interest rate as a decimal
n = number of compounding periods in 1 year

This is useful because it requires only the nominal interest rate and the number of compounding periods in a year. We do not need to know the principal, nor do we need to calculate the amount of interest earned in the first year.

Worked Example 9

WE 9

A loan attracts an interest rate of 23% compounded daily. Find:

(a) the nominal rate, R, and the number of compounding periods, n

(b) the effective rate, R_{ef}, using the values found in **(a)**.

Thinking	Working
(a) 1 The nominal rate is the stated rate.	(a) $R = 23\%$
2 The number of compounding periods will be the number of days in a year.	$n = 365$
(b) 1 Convert the nominal rate to a decimal.	(b) $R = 23\% = 0.23$
2 Write down the formula.	$R_{ef} = \left[\left(1 + \frac{R}{n}\right)^n - 1\right] \times 100$
3 Substitute the decimal value of R and the value of n into the formula.	$R_{ef} = \left[\left(1 + \frac{0.23}{365}\right)^{365} - 1\right] \times 100$
4 Calculate R_{ef} as a percentage.	$= 0.2585 \times 100$ $= 25.85\%$ p.a.

5 Ned needs to buy a new laptop. He is planning to take out a personal loan with an interest rate of 20.5% p.a. compounded quarterly. The alternative is to use his credit card that has an interest rate of 20.1% compounded daily.

(a) What is the effective rate of this personal loan?

(b) What is the credit card's effective rate?

(c) Which of the two options should Ned choose and why?

6 A bed can be purchased on a 'get it now, pay in 12 months' plan.

The interest payment options are different depending on whether the purchase is made online or instore. The online purchase attracts a flat rate interest of 25% p.a. The interest on the instore purchase is 23% p.a. compounded daily.

(a) What is the real or effective interest rate charged on an instore purchase?

(b) Which is the cheaper option?

Reasoning

7 The following represent investment options. In each case, choose the better option *without calculating the effective rate* and justify your choice.

(a) 12% p.a. compounded monthly
15% p.a. compounded daily

(b) 10.5% p.a. compounded weekly
10.5% p.a. compounded monthly

(c) 11% p.a. compounded daily
12% p.a. compounded daily

(d) 15% p.a. flat rate for 2 years
15% p.a. compounded annually for 2 years

8 The options in Question **7** now represent loans. Will your choice of option change? Justify your answer.

9 Inez and Sam need to choose between the following lucrative investments:

Option A yields 35.2% p.a. compounded daily.
Option B yields 42% p.a. compounded annually.
Option C yields 40% p.a. compounded quarterly.

Which option should they choose and why?

10 Simon has saved the 10% deposit for a new car. The car company can offer him an interest-free loan on the remaining balance for 6 months and then 15% p.a. compounded daily. Alternatively, he could borrow the money elsewhere for a flat rate of 9% p.a.

(a) If he pays out the loan plus interest in a year, which option will be cheaper?

(b) If he pays out the loan plus interest in two years, which option will be cheaper?

Open-ended

11 'Nominal interest rate' and 'simple interest rate' are the same thing. Explain with an example why this statement is untrue.

12 Two investments have different nominal interest rates and different compounding periods. However, the effective interest rate for both is exactly 46.41% p.a. What might be the nominal rates and respective compounding periods for these two investments?

(b) No

(c) (i) $I \approx 16\%$ of P (ii) $I \approx 14\%$ of P (iii) $I \approx 13\%$ of P

Open-ended – Sample answers

10 The statement means that if compound interest is calculated more frequently (e.g. every month instead of every quarter), it will result in a greater amount of interest. Quarterly compounding gives interest of \$4859.47. (48.6%) Monthly compounding gives interest of \$4898.46. (49.0%) Weekly compounding gives interest of \$4913.66. (49.1%)

11 Using the suggested example, simple interest is \$600 and interest compounded annually is \$630.50. The principal gets bigger after each time period.

Exercise 1.3 (p. 23)

1 (a) \$5996 (b) \$5437 (c) \$5456.76 (d) B

2 (a) 10.2% p.a. (b) 3.24% p.a. (c) 2.6% p.a. (d) B

3 (a) 12 years (b) 10 years (c) 4.2 years

4 (a) \$5956.42 (b) \$956.42

(c) only marginally better, I = \$956.46

(d) \$956.42 (e) 4.47% p.a.

(f) At least 4 years, as it would be worth \$5788.13 at the end of the third year and \$6077.53 at the end of the fourth year.

5 2

6 (a) every 4 months (b) 9.3% p.a.

(c) monthly compounding

7 At least every 4 months.

Open-ended – Sample answers

8 Rounding error: $\frac{0.065}{4} = 0.01625$, $P = \$18\,751.14$

9 11% p.a. with annual compounding, 10.5% p.a. compounding half-yearly, 10.2% p.a. compounding daily

Half-time 1 (p. 26)

1 B 2 \$8673.33 3 \$8889.26 4 200%

5 (a) \$50 649.49 (b) \$51 824.82 (c) \$52 059.90

6 9

7 (a) \$10 866 (b) \$9000

(c) Compound interest is \$1866 more than simple interest.

8 17.27% p.a.

Exercise 1.4 (p. 30)

1 (a) 7% p.a. (b) 9.4% p.a. (c) 13.52% p.a.

(d) 21.73% p.a. (e) 17.88% p.a. (f) 6.45% p.a.

2 (a) (i) $R = 7.3\%$ p.a., $n = 5$ (ii) 7.52% p.a.

(b) (i) $R = 15.5\%$ p.a., $n = 12$ (ii) 16.65% p.a.

(c) (i) $R = 21\%$ p.a., $n = 4$ (ii) 22.71% p.a.

(d) (i) $R = 16\frac{1}{4}\%$ p.a., $n = 3$ (ii) 17.15% p.a.

(e) (i) $R = 20\frac{1}{3}\%$ p.a., $n = 365$ (ii) 22.54% p.a.

(f) (i) $R = 5.25\%$ p.a., $n = 2$ (ii) 5.32% p.a.

3 B

4 (a) Q Bank: 15.87% p.a., D Bank: 16.06% p.a.

(b) D Bank

5 (a) 22.13% p.a. (b) 22.26% p.a.

(c) Personal loan because the effective rate is lower.

6 (a) 25.85% p.a. (b) online

7 (a) 15% p.a. compounded daily—higher rate and more frequent compounding

(b) 10.5% p.a. compounded weekly—rate is the same so more frequent compounding is better

(c) 12% p.a. compounded daily—compounding frequency is the same so higher rate is better

(d) 15% p.a. compounded annually for 2 years—compounded beats flat rate over 2 years

8 Yes. In each case choose the other option because for loans we want the lowest effective rate.

9 Option C has the highest effective rate but all the options look too good to be true.

10 (a) car company loan (b) flat rate

Open-ended – Sample answers

11 1-year loan of \$10 000 at 6% p.a. compounded daily. Nominal rate is 6% p.a. The interest is compounded, not simple. However, the same amount of interest will be paid if \$10 000 is borrowed for a year at a simple interest rate of 6.18% p.a. Either way nominal isn't the same as simple.

12 42% p.a. compounded half-yearly, 40% p.a. compounded quarterly

Exercise 1.5 (p. 39)

1 (a) (i) \$10 500 (ii) \$19 500

(b) (i) \$28 500 (ii) \$19 000

(c) (i) \$100 000 (ii) No value

(d) (i) \$772 225 (ii) \$17 775

(e) (i) \$60 900 (ii) \$84 100

(f) (i) \$107 250 (ii) \$192 750

(g) (i) \$4928.13 (ii) \$42 571.87

(h) (i) \$76 159.13 (ii) \$76 540.87

2 (a) (i) \$20 870.65 (ii) \$9 129.35

(b) (i) \$25 667.10 (ii) \$21 832.90

(c) (i) \$35 848.59 (ii) \$64 151.41

(d) (i) \$290 950.49 (ii) \$499 049.51

(e) (i) \$93 037.95 (ii) \$51 962.05

(f) (i) \$207 293.23 (ii) \$92 706.77

(g) (i) \$42 285.67 (ii) \$5214.33

(h) (i) \$83 275.25 (ii) \$69 424.75

3 (a) (i) \$6000 (ii) \$2/hour

(iii) \$20 (iv) \$6130